

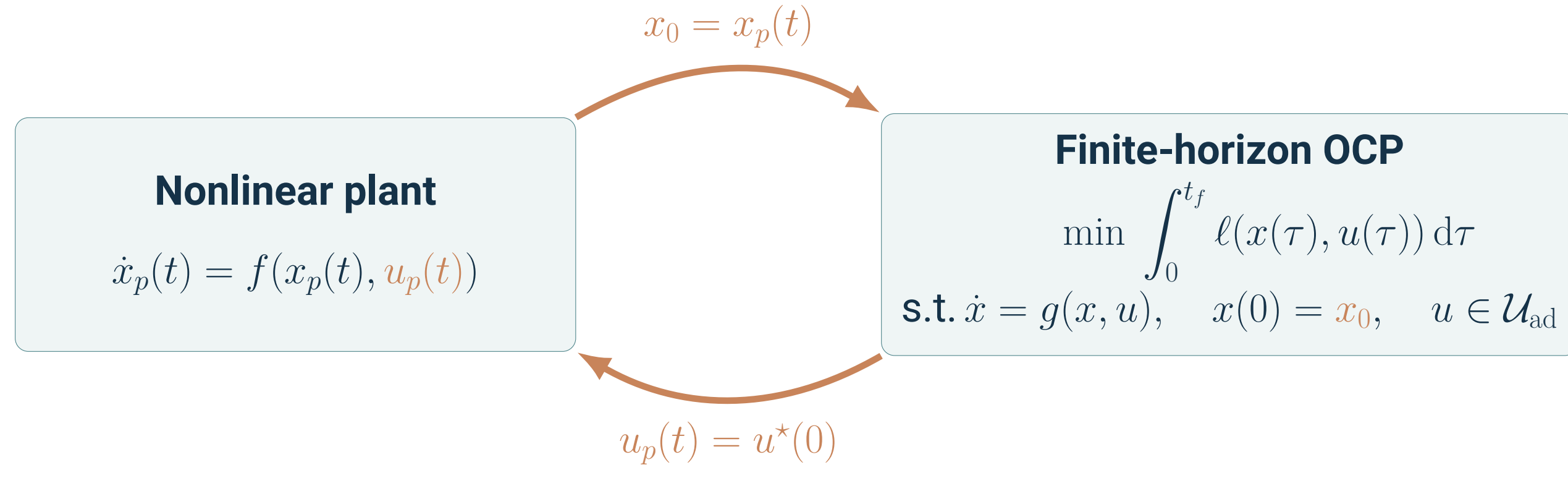
Optimization-Based Control by Interconnection of Nonlinear Port-Hamiltonian Systems

Till Preuster

Funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation)
– Project-ID 531152215 – CRC 1701 “Port-Hamiltonian Systems”

MPC motivation

MPC: one full OCP solve in each implementation loop



Challenge: solve a nonlinear, high-dimensional optimality system per feedback step

$$\min_{(x,u) \in H^1(0,t_f;\mathbb{R}^n) \times L^2(0,t_f;\mathbb{R}^m)} \int_0^{t_f} \ell(x(\tau)) + \frac{\alpha}{2} \|u(\tau)\|_{\mathbb{R}^m}^2 d\tau$$

s.t. $\dot{x} = Ax + Bu, \quad x(0) = x_0, \quad u(\tau) \in U_{\text{ad}} \text{ a.e.}$

Let $\mathcal{Y} := H^1(0, t_f; \mathbb{R}^n)$, $\mathcal{X} := L^2(0, t_f; \mathbb{R}^n)$, $\mathcal{U} := L^2(0, t_f; \mathbb{R}^m)$.
(solution space) (control space)

Operator form: write constraint as operator equation

$$\underbrace{\begin{bmatrix} \frac{d}{d\tau} - A \\ \delta_0 \end{bmatrix}}_{=: \mathcal{A}} x - \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{=: \mathcal{B}} u = \begin{bmatrix} 0 \\ x_0 \end{bmatrix}, \quad u \in \mathcal{U}_{\text{ad}},$$

where

- i) $\mathcal{A} : \mathcal{X} \supset H^1(0, t_f; \mathbb{R}^n) \rightarrow \mathcal{Z} := \mathcal{X} \times \mathbb{R}^n, \mathcal{B} \in L(\mathcal{U}, \mathcal{Z});$
- ii) $J(x) := \int_0^{t_f} \ell(x(\tau)) d\tau$ convex, Fréchet differentiable;
- iii) convex and closed admissible control set

$$\mathcal{U}_{\text{ad}} := \{u \in \mathcal{U} : u(\tau) \in U_{\text{ad}} \text{ for a.e. } \tau \in (0, t_f)\} \neq \emptyset.$$

Theorem (Optimal solutions)

There exists unique solution $(x^*, u^*) \in \text{dom } \mathcal{A} \times \mathcal{U}$ of **[OCP]**.
Moreover, there is $p^* \in \text{dom } \mathcal{A}^*$ such that (x^*, p^*) solves

$$(1) \quad \mathcal{S}(x, p) := \begin{bmatrix} 0 & -\mathcal{A}^* \\ \mathcal{A} & 0 \end{bmatrix} \begin{bmatrix} x \\ p \end{bmatrix} - \begin{bmatrix} \nabla J(x) \\ \mathcal{B} P_{\mathcal{U}_{\text{ad}}}(\frac{1}{\alpha} \mathcal{B}^* p) \end{bmatrix} = \begin{bmatrix} 0 \\ \begin{bmatrix} 0 \\ x_0 \end{bmatrix} \end{bmatrix}.$$

For $p = (\lambda, \lambda_0)$, the optimal feedback is $u^*(0) = P_{U_{\text{ad}}}(\frac{1}{\alpha} B^\top \lambda_0^*)$.

The optimizer is a port-Hamiltonian system

Primal-dual gradient flow in algorithmic time s , $\mathcal{W} := [\mathcal{X} \mathcal{Z}]$:

$$(2) \quad \begin{bmatrix} \dot{x}(s) \\ \dot{p}(s) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -\mathcal{A}^* \\ \mathcal{A} & 0 \end{bmatrix}}_{=: \mathcal{J} \text{ (skew-adjoint)}} \begin{bmatrix} x(s) \\ p(s) \end{bmatrix} - \underbrace{\begin{bmatrix} \nabla J(x(s)) \\ \mathcal{B} P_{\mathcal{U}_{\text{ad}}}(\frac{1}{\alpha} \mathcal{B}^* p(s)) \end{bmatrix}}_{=: \mathcal{R}(x(s), p(s)) \text{ (monotone)}} - \begin{bmatrix} 0 \\ \begin{bmatrix} 0 \\ x_0 \end{bmatrix} \end{bmatrix}$$

Theorem (Exponential optimizer convergence)

Let $w^* := (x^*, p^*)$ be the unique solution of (1) and

$$\mu \|x - y\|_{\mathcal{X}}^2 \leq \langle \nabla J(x) - \nabla J(y), x - y \rangle_{\mathcal{X}} \leq L \|x - y\|_{\mathcal{X}}^2.$$

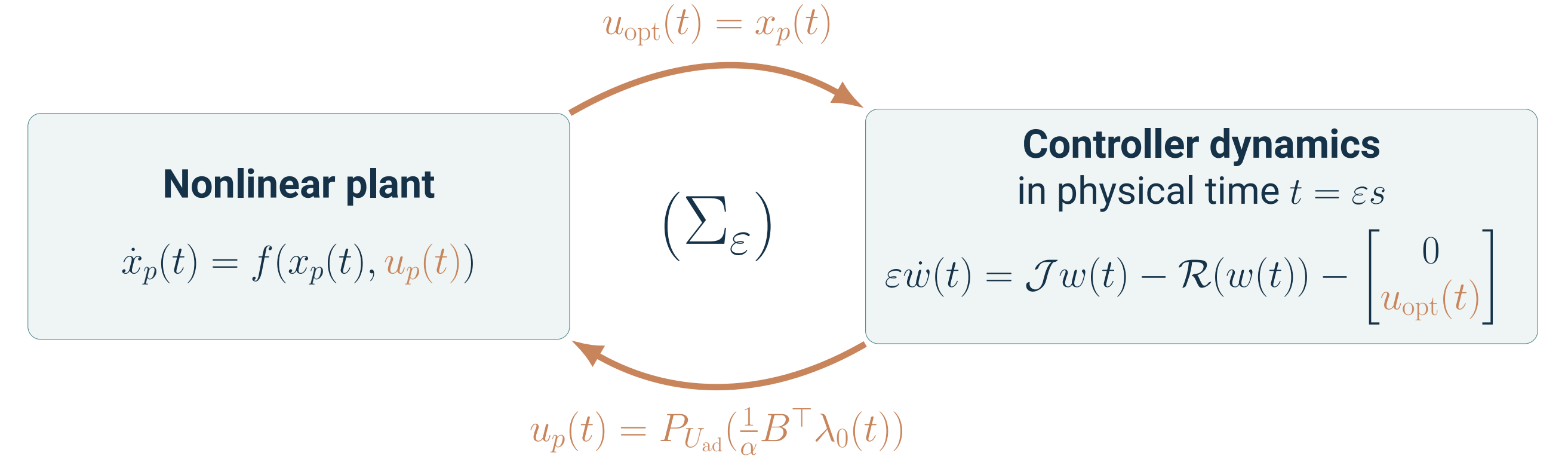
Then, w^* is globally uniformly exponentially stable for (2):

$$\text{for } M \geq 1, \eta > 0 : \quad \|w(s, w_0) - w^*\|_{\mathcal{W}} \leq M e^{-\eta s} \|w_0 - w^*\|_{\mathcal{W}}.$$

Proof idea: Lyapunov functional with cross term:

$$W_\rho(x, p) := \frac{1}{2} (\|x\|_{\mathcal{X}}^2 + \|p\|_{\mathcal{Z}}^2) + \rho \langle x, \mathcal{A}^{-1} p \rangle_{\mathcal{X}}$$

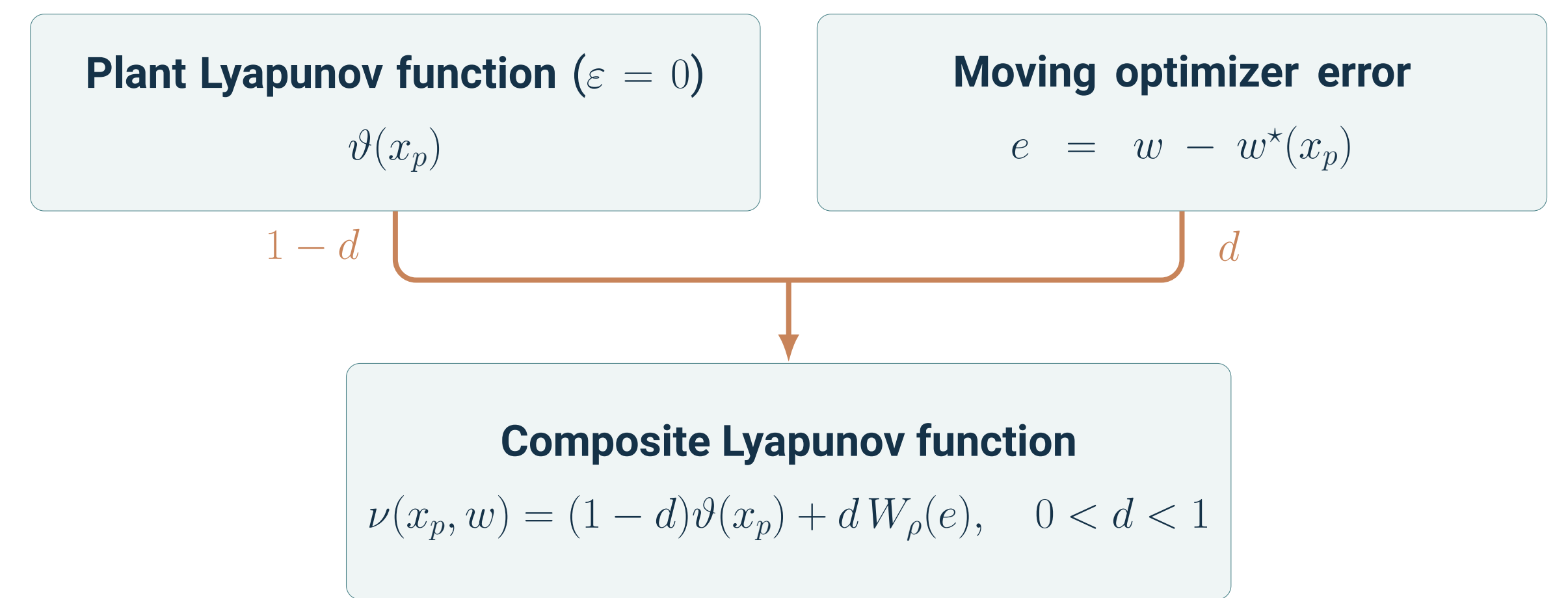
Replace each exact solve by a dynamic controller



$\varepsilon = 0$: optimizer jumps to $w^* = w^*(x_p)$ (nominal MPC)

$\varepsilon > 0$: optimizer approximates $w^*(x_p)$ only (subopt. MPC)

If MPC stabilizes, so does the running optimizer



Theorem (Closed-loop stability)

Assume that 0 is a locally asymptotically stable equilibrium of (Σ_ε) for $\varepsilon = 0$ (nominal MPC works). Then, there exists $\varepsilon^* > 0$ such that this stability persists for all $\varepsilon \in (0, \varepsilon^*)$.

PROOF

=

SINGULAR PERTURBATION
ARGUMENTS

+

EXPONENTIAL DECAY
OF THE OPTIMIZER

Optimal control problem

Optimizer dynamics

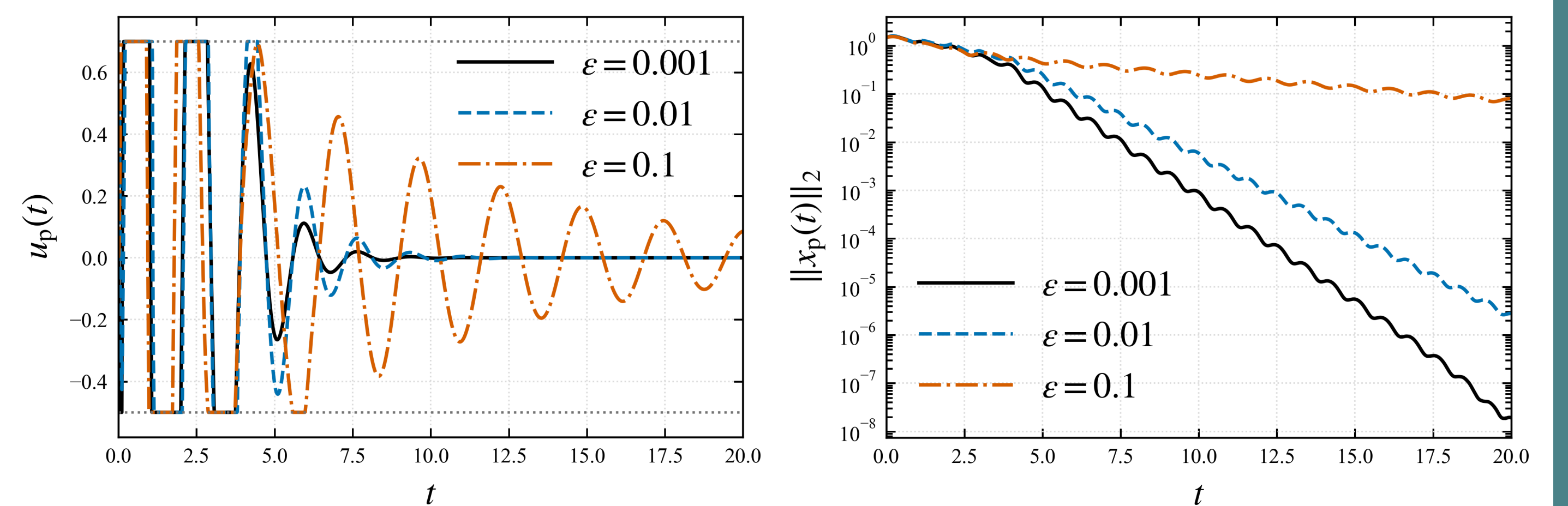
Nonlinear plant, linear prediction model

Plant: Duffing oscillator $\dot{q} = p, \dot{p} = -q - \beta q^3 + u_p$

Prediction model: linearization ($\beta = 0$):

$$\min_{(x,u) \in H^1(0,5;\mathbb{R}^2) \times L^2(0,5)} \int_0^5 \frac{1}{2} \|x(\tau)\|^2 + \frac{\alpha}{2} |u(\tau)|^2 d\tau$$

$$\text{s.t. } \dot{x}(\tau) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x(\tau) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(\tau), \quad u(\tau) \in [-0.5, 0.7].$$



Control-by-Interconnection

Stability of closed-loop system

Numerical example

References



Preuster, Gernandt, and Schaller. “Optimization-based control by interconnection of nonlinear port-Hamiltonian systems”. *arXiv:2602.06670* (2026).



Preuster, Gernandt, and Schaller. “Coupling optimization algorithms and monotone control systems: suboptimal model predictive control as an operator splitting scheme”. *arXiv:2605.23581* (2026).



Preuster, Gernandt, and Schaller. “Stabilization of monotone control systems with input constraints”. *arXiv:2603.07763* (2026).



Pham, Vu, Prodan, and Lefèvre. “A combined Control by Interconnection-Model Predictive Control design for constrained Port-Hamiltonian systems”. *Systems & Control Letters* (2022).



Stegink, De Persis, and van der Schaft. “Port-Hamiltonian formulation of the gradient method applied to smart grids”. *IFAC-PapersOnLine* 48(13) (2015), pp. 13–18.