Optimal allocation strategies of perennial plants

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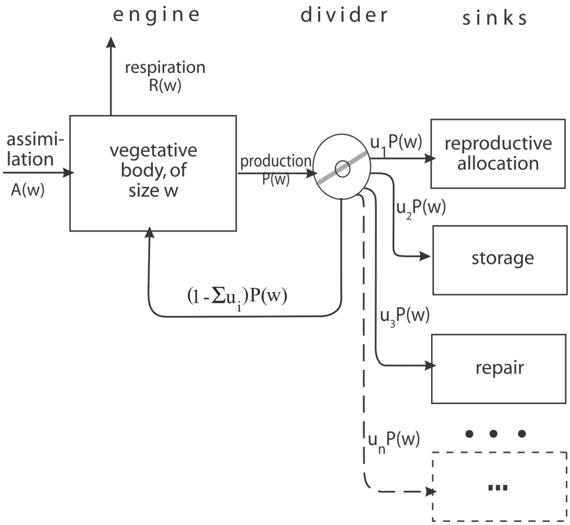
Outline

The paradigm of optimal allocation models



OAM of perennial plants

Common approach

- The whole life is divided into discrete periods with favorable conditions
- Within the period the problem is solved via PMP
- Then OA problem is solved for all life via dynamic programming

Disadvantages of a common approach

- In the above models annuality and perenniality are assumptions
- Complicated life-histories during favourable seasons contrast to the simple jump of a state during unfavourable seasons.

Our aims

- To derive a model, which will encompass
  1. annuals
  2. monocarpic perennials
  3. evergreen perennials
  4. polycarpic plants

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- To derive a model, which will encompass
  1. annuals
  2. monocarpic perennials
  3. evergreen perennials
  4. polycarpic plants
- Investigate life-strategies during unfavourable periods
- Provide sufficient conditions for monocarpic strategy
- Connect optimal allocation models and theory of trade-offs between size/number of seeds

Continuous-time model of a perennial plant

Compartments of a plant

- $x_1(t)$ is a mass of vegetative compartment at time t,
- $x_2(t)$ is a mass of reproductive compartment at time t,
- $x_3(t)$ is a mass of nonstructural carbohydrates at time t.

Controls

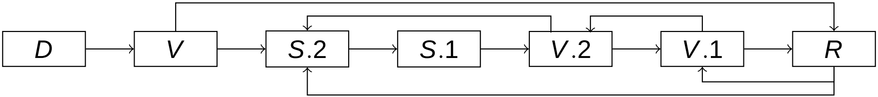
- $v(t) \in [0,1]$ - total allocation rate
- $v_1(t) \in [0, v(t)]$ - allocation rate to vegetative tissues

$$\begin{aligned}
\dot{x}_1 &= v_1(t)g(x_3) - \mu(t)x_1, \\
\dot{x}_2 &= (v(t) - v_1(t))g(x_3), \\
\dot{x}_3 &= \zeta(t)f(x_1) - v(t)g(x_3) - \omega(t)x_3.
\end{aligned}$$

Goal: Maximization of fitness

$$\int_{t_0}^{T} L(s)\dot{x}_2(s)ds = \int_{t_0}^{T} L(s)(v(s) - v_1(s))g(x_3(s))ds \to \max.$$

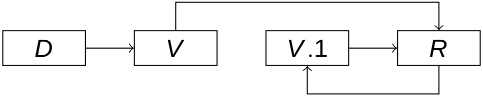
Stages of perennial plant development

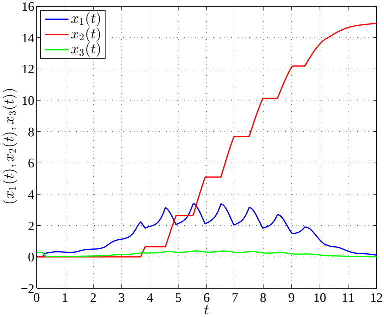


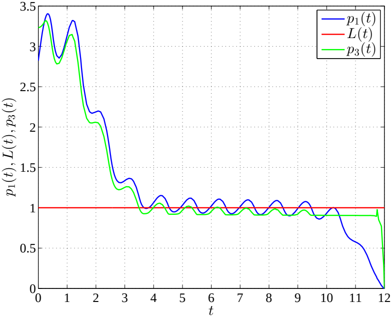
1. Vegetative period: $p_1 > \max\{L, p_3\}$ $\Rightarrow v = v_1 = 1.$
2. Reproductive period: $L > \max\{p_1, p_3\}$ $\Rightarrow v = 1, v_1 = 0.$
3. Storage period: $p_3 > \max\{p_1, L\}$ $\Rightarrow v = v_1 = 0.$

- V – vegetative growth after germination
- $S.2$ – preparing for unfavourable climate conditions
- $S.1$ – life in unfavourable climatic conditions
- $V.2$ – vegetative period
- $V.1$ – allocation to vegetative tissues before reproduction
- R – reproductive allocation

Annual plant with multiple reproduction periods

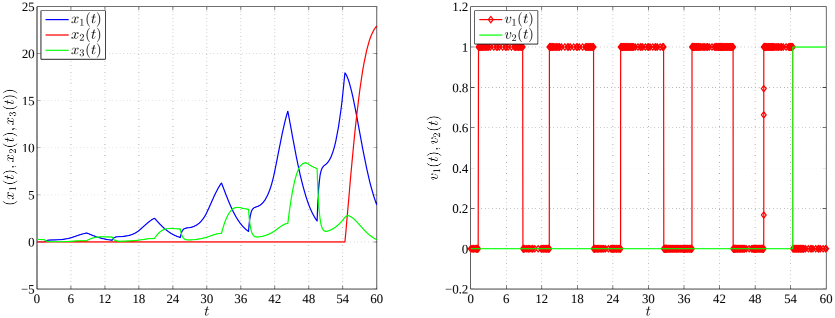






Multiple reproduction periods appear due to losses of vegetative mass, caused by external factors $\mu(\cdot)$.

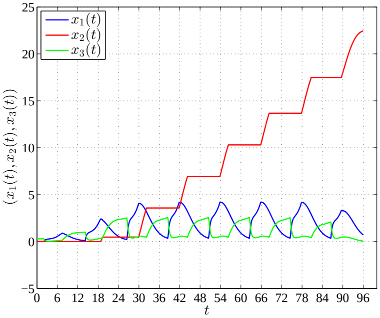
Monocarpic plants

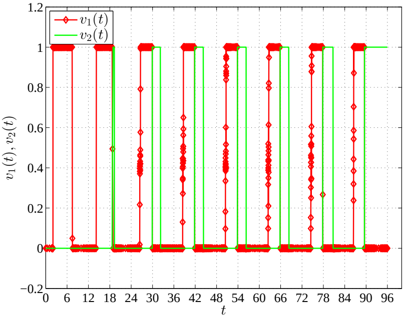


Proposition

If survivability $L \equiv const$ and storage losses $\omega \equiv 0$, then the plant is monocarpic.

Polycarpic perennial





Optimization of a mass of a seed

Main assumptions

- Fitness of the parent is equal to the sum of fitnesses of all its descendants
- The probability of germination does not depend on the size of the seed.

Equations, governing the dynamics of a plant

$$\begin{cases} \dot{x}_1 = v_1(t)g(x_3) - \mu(t)x_1, \\ \dot{x}_2 = (v(t) - v_1(t))g(x_3), \\ \dot{x}_3 = \zeta(t)f(x_1) - v(t)g(x_3) - \omega(t)x_3, \\ x(0) = \frac{1}{a}y_0. \end{cases}$$

Maximization of fitness

$$\max_{0\leq v(t)\leq 1,\ 0\leq v_1(t)\leq v(t),\ a\in[1,\infty)} Q_a = a\int_{t_0}^{T} L(s)\dot{x}_2(s)ds,$$

Concavity of photosynthetic rate

Theorem

Let f, g be concave functions, $f(0) = g(0) = 0$. Then a plant should produce as much seeds as possible.

The sense of concavity

Efficiency of photosynthesis declines with the growth of a plant

- Self-shading
- boundedness of resources
- etc.

Some consequences

- colonizing species
- plants, living in open environments

should have small seeds

Convexity of photosynthetic rate

Theorem

Let f, g be convex functions, $f(0) = g(0) = 0$. Then the size of the seeds should be as large as possible.

Above theorem implies:

- In the closed and shady environments
- under mineral shortage
- if there is a strong competition with the established vegetation

The size of seeds cannot be too small

If f and g are linear functions then all partition strategies are equivalent!

Conclusion

- A continuous-time OAM of a perennial plant has been developed
- The model can be used for modeling of
  1. annual plants with single/multiple reproduction periods
  2. monocarpic perennials
  3. evergreen perennials
  4. polycarpic perennials
- Within OAM framework size-number (of seeds) trade-offs have been analysed

Outlook

- Why are some perennials monocarpic?
- What is appropriate measure of fitness: lifetime offspring production or lifetime offspring production by this plant and its descendants or ...?
- How to model plants with vegetative reproduction?
- How to model life-strategies of a plant in a non-native environment?

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Thank you for attention!