Optimal allocation strategies of perennial plants

Andrii Mironchenko\textsuperscript{1} \hspace{1cm} Jan Kozłowski\textsuperscript{2}

\textsuperscript{1}Institute of Mathematics, University of Würzburg, Germany
\textsuperscript{2}Institute of Environmental Sciences, Jagiellonian University, Poland

December 11, 2013
52nd IEEE Conference on Decision and Control
Florence, Italy
1. Optimal allocation models (OAM)

2. Optimal seed size

3. Conclusion
The paradigm of optimal allocation models

A. Mironchenko, J. Kozłowski

Optimal allocation strategies of perennial plants
Common approach

- The whole life is divided into discrete periods with favorable conditions
- Within the period the problem is solved via PMP
- Then OA problem is solved for all life via dynamic programming

Disadvantages of a common approach

- In the above models annuality and perenniality are assumptions
- Complicated life-histories during favourable seasons contrast to the simple jump of a state during unfavourable seasons.
Our aims

To derive a model, which will encompass:

1. annuals
2. monocarpic perennials
3. evergreen perennials
4. polycarpic plants
Our aims

- To derive a model, which will encompass:
  1. annuals
  2. monocarpic perennials
  3. evergreen perennials
  4. polycarpic plants
- Investigate life-strategies during unfavourable periods
- Provide sufficient conditions for monocarpic strategy
- Connect optimal allocation models and theory of trade-offs between size/number of seeds
Continuous-time model of a perennial plant

Compartments of a plant

- $x_1(t)$ is a mass of vegetative compartment at time $t$,
- $x_2(t)$ is a mass of reproductive compartment at time $t$,
- $x_3(t)$ is a mass of nonstructural carbohydrates at time $t$.

Controls

- $v(t) \in [0, 1]$ - total allocation rate
- $v_1(t) \in [0, v(t)]$ - allocation rate to vegetative tissues

\[
\begin{align*}
\dot{x}_1 &= v_1(t)g(x_3) - \mu(t)x_1, \\
\dot{x}_2 &= (v(t) - v_1(t))g(x_3), \\
\dot{x}_3 &= \zeta(t)f(x_1) - v(t)g(x_3) - \omega(t)x_3.
\end{align*}
\]

Goal: Maximization of fitness

\[
\int_{t_0}^{T} L(s)\dot{x}_2(s)ds = \int_{t_0}^{T} L(s)(v(s) - v_1(s))g(x_3(s))ds \rightarrow \max.
\]
Stages of perennial plant development

1. Vegetative period: $p_1 > \max\{L, p_3\}$ \implies v = v_1 = 1.
2. Reproductive period: $L > \max\{p_1, p_3\}$ \implies v = 1, v_1 = 0.
3. Storage period: $p_3 > \max\{p_1, L\}$ \implies v = v_1 = 0.

- V – vegetative growth after germination
- S.2 – preparing for unfavourable climate conditions
- S.1 – life in unfavourable climatic conditions
- V.2 – vegetative period
- V.1 – allocation to vegetative tissues before reproduction
- R – reproductive allocation

A. Mironchenko, J. Kozlowski
Optimal allocation strategies of perennial plants
Annual plant with multiple reproduction periods

Multiple reproduction periods appear due to losses of vegetative mass, caused by external factors $\mu(\cdot)$.
Proposition

If survivability $L \equiv \text{const}$ and storage losses $\omega \equiv 0$, then the plant is monocarpic.
Polycarpic perennial

A. Mironchenko, J. Kozlowski

Optimal allocation strategies of perennial plants
Optimization of a mass of a seed

Main assumptions

- Fitness of the parent is equal to the sum of fitnesses of all its descendants
- The probability of germination does not depend on the size of the seed.

Equations, governing the dynamics of a plant

\[
\begin{align*}
\dot{x}_1 &= v_1(t)g(x_3) - \mu(t)x_1, \\
\dot{x}_2 &= (v(t) - v_1(t))g(x_3), \\
\dot{x}_3 &= \zeta(t)f(x_1) - v(t)g(x_3) - \omega(t)x_3, \\
x(0) &= \frac{1}{a}y_0.
\end{align*}
\]

Maximization of fitness

\[
\max_{0 \leq v(t) \leq 1, \, 0 \leq v_1(t) \leq v(t), \, a \in [1, \infty)} Q_a = a \int_{t_0}^{T} L(s)\dot{x}_2(s)ds,
\]
### Concavity of photosynthetic rate

**Theorem**

Let \( f, g \) be concave functions, \( f(0) = g(0) = 0 \). Then a plant should produce as much seeds as possible.

**The sense of concavity**

Efficiency of photosynthesis declines with the growth of a plant
- Self-shading
- boundedness of resources
- etc.

**Some consequences**

- colonizing species
- plants, living in open environments

should have small seeds
**Theorem**

Let $f, g$ be convex functions, $f(0) = g(0) = 0$. Then the size of the seeds should be as large as possible.

**Above theorem implies:**

- In the closed and shady environments
- under mineral shortage
- if there is a strong competition with the established vegetation

The size of seeds cannot be too small

If $f$ and $g$ are linear functions then all partition strategies are equivalent!
A continuous-time OAM of a perennial plant has been developed.
The model can be used for modeling of:
1. annual plants with single/multiple reproduction periods
2. monocarpic perennials
3. evergreen perennials
4. polycarpic perennials

Within OAM framework size-number (of seeds) trade-offs have been analysed.
Outlook

- Why are some perennials monocarpic?
- What is appropriate measure of fitness: lifetime offspring production or lifetime offspring production by this plant and its descendants or ...?
- How to model plants with vegetative reproduction?
- How to model life-strategies of a plant in a non-native environment?
Why are some perennials monocarpic?

What is appropriate measure of fitness: lifetime offspring production or lifetime offspring production by this plant and its descendants or ...?

How to model plants with vegetative reproduction?

How to model life-strategies of a plant in a non-native environment?

Journal version and slides can be found on www.mironchenko.com
Outlook

- Why are some perennials monocarpic?
- What is appropriate measure of fitness: lifetime offspring production or lifetime offspring production by this plant and its descendants or ...?
- How to model plants with vegetative reproduction?
- How to model life-strategies of a plant in a non-native environment?

Journal version and slides can be found on [www.mironchenko.com](http://www.mironchenko.com)

Thank you for attention!